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LAMINAR FREE CONVECTION FROM A VERTICAL PLATE

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Abstract—An asymptotic solution of the Navier–Stokes and energy equations for the problem of free convection from a vertical finite plate is considered. Boundary-layer interaction with the external flow is studied by the method of matched asymptotic expansions. The effect of the leading edge has been analysed with the aid of a deformed longitudinal coordinate. The flow structure and heat transfer in the vicinity of the trailing edge of the plate are studied within the confines of the two-layer boundary-layer model. It is found that the wake effect on the boundary layer at the plate is manifested through the external flow. The solution results have been supported by experimental data on the velocity and temperature profiles and on heat transfer coefficients both in the case of the first-kind boundary conditions for the plate and in the case of constant heat flux.

NOMENCLATURE

a	thermal diffusivity
Gr	Grashof number, $g\beta\Delta TL^3/\nu^2$
Gr_x	local Grashof number
Gr^*	modified Grashof number
Gr_x^*	local modified Grashof number
g	gravity acceleration
L	characteristic length
Nu	Nusselt number, $\alpha L/\lambda$
Nu_x	local Nusselt number, $\bar{\alpha}\bar{x}/\lambda$
Pr	Prandtl number, ν/a
q_w	heat flux on the plate
r	$(x^2+y^2)^{1/2}$
T	temperature
T_∞	ambient temperature
T_w	plate temperature
ΔT	excess temperature, $T_w - T_\infty$
u	$\bar{u}/(g\beta\Delta TL)^{1/2}$
$\bar{u}$	longitudinal velocity
$\bar{v}$	transverse velocity
x, y	dimensionless coordinates, $\bar{x}/L$ and $\bar{y}/L$, respectively
$\bar{x}$	longitudinal coordinate
$\bar{y}$	transverse coordinate.

Greek symbols

$\bar{\alpha}$	heat transfer coefficient
β	thermal coefficient of expansion
η	similarity variable
λ	thermal conductivity
ν	kinematic viscosity
ϕ	arctan y/x
Ψ	stream function
∇^2	Laplace operator.

1. INTRODUCTION

THE DESCRIPTION of free convective flow and heat transfer from a vertical plate is one of the main model problems of free convection. The investigation of the specific features of physical processes occurring in the course of free convection near a vertical plate and their adequate mathematical interpretation favour the understanding of the laws governing convective heat transfer in other, more complex, cases. The theory of free convection can be considered as an integral part of the boundary-layer theory.

This exceptional situation with free convection about a vertical plate has been investigated in detail experimentally and theoretically [1, 2]. However, even now these investigations cannot be regarded as completed. Different correlations [3–6] suggested for the calculation of heat transfer, which leave the physics of the phenomenon aside, only attest to the above.

Initial successful attempts to describe free convection from a vertical plate within the confines of the classical boundary-layer theory were attributed to the similarity character of its solutions. The significance of similarity is due not only to the fact that the reduction to a set of ordinary differential equations allows one to obtain the solution and, simultaneously, the qualitative understanding of the phenomenon. The similarity provides an intermediate asymptotic representation of the solutions of substantially more extensive classes of problems [7]. The boundary-layer theory solutions are applicable only far from the leading and trailing edges and other geometric inhomogeneities; they describe the flow only near the surface and, these conditions being fulfilled, have a universal nature. It is this universality which explains why in the early studies of free convection the experimental data on heat transfer from vertical and horizontal plates, vertical and horizontal cylinders were interpreted by a single curve of the boundary-layer theory.

The disadvantages of the boundary-layer theory are the continuation of its advantages. Being restricted to the description of the flow in a narrow wall zone, this theory ignores the origination of an external flow. In a neighbourhood of the discontinuity points of the boundary conditions, the boundary-layer theory breaks down because of the parabolicity and asymptotic nature of its equations. These very reasons prevent the study of the reverse effect of the wake on the boundary layer near the plate.

Accurate measurements of the velocity and temperature fields in free convection from a vertical plate [8–10] reveal an appreciable and systematic deviation of the experimental data from the boundary-layer theory curves. Therefore, it is necessary to use a complete set of the Navier–Stokes and energy equations to more accurately describe free convection.

Getting out of the framework of the classical boundary-layer theory is associated with the development of singular perturbation methods [11–13]. A successive application of the method of matched asymptotic expansions to the problems of free convection near a vertical plate allowed one to construct the solution to the Navier–Stokes and energy equations up to and including the second approximation [14–17] and to study the interaction of the boundary layer with the external flow. The only drawback of these investigations is the appearance of eigensolutions in the inner expansion and the impossibility to compare predicted results with experimental data because of indeterminacies of corresponding constants.

The effect of the leading edge on free-convective heat transfer is attributed to the upstream induced flow which is observed experimentally [18]. The possibility of interpreting the effect of the leading edge by varying a relative vertical position of the boundary layer was considered in ref. [19]. The results obtained within the confines of the boundary-layer theory make it impossible to derive the quantitative characteristics of the effect studied. The major difficulty in the study of the leading edge effects resides in the discrepancy between the Grashof number, which determines the nature of the process as a whole, and its local value in the vicinity of the leading edge. A qualitative assessment of the scale of the leading edge neighbourhood, where the boundary-layer approximation proves its undoing, has been carried out in ref. [20]. A detailed description of motion near the leading edge within the framework of the asymptotic theory at large Grashof numbers is impossible and is hardly necessary. It is only important to know the overall effect exhibited by this motion on the flow and heat transfer in the boundary layer.

The structure of the flow in the vicinity of the trailing edge of a vertical plate, analysed in ref. [20] and based on the solution of the problems of streamlining [21–24], has been actually described within the framework of the boundary-layer theory and does not account for the specific features of free convection.

The reverse effect of the wake on the boundary layer near a vertical plate was first studied in ref. [25] on the assumption of parallel streamlines in the wake. Investigation of first-order perturbations in the presence of such a specific wake resulted in a decrease of heat transfer contrary to what was observed experimentally. Explaining the result obtained by the impossibility to account for the leading edge effect, the authors attempted a separate study of flow in the neighbourhood of the leading edge [26]. The arbitrariness in the choice of the leading edge effect scale did not allow an effective comparison between the predicted and experimental results. Direct numerical calculations of the system of Navier–Stokes and energy equations were of little value also [27].

Thus, the main sources of difficulties encountered in the theoretical description of free convection from a vertical plate are the following factors:

(1) the leading edge effects;
(2) the flow at the trailing edge;
(3) the effect of the wake behind the plate.

The encouraging results obtained using the methods of singular perturbations in the study of free convection boundary-layer interaction with the external flow have drawn special attention to these methods.

This paper concerns itself with the development of the methods of singular perturbations in an effort to theoretically describe free convection from a vertical plate with regard for the above factors. The merits of these methods lie in the general approach to the study of separate problems, the simplicity of formalism and the clarity of the physical meaning of the results obtained [28].

2. STATEMENT OF THE PROBLEM

The initial system of equations to describe free convection from a vertical plate is taken to be the system of Navier–Stokes and energy equations. For a two-dimensional (2-D) developed flow this system is written down in dimensionless form in terms of the stream function and the excess temperature as [28, 29]

$$\frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\Psi)-\frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\Psi) = Gr^{-1/2}\,\nabla^4\Psi+\frac{\partial \Theta}{\partial y},$$

$$\frac{\partial \Psi}{\partial y}\frac{\partial \Theta}{\partial x}-\frac{\partial \Psi}{\partial x}\frac{\partial \Theta}{\partial y} = Pr^{-1}\,Gr^{-1/2}\,\nabla^2\Theta. \quad (1)$$

The x-axis is directed along the plate from the leading edge, the y-axis is normal to it. The physical properties of the medium (except the density) are assumed to be constant. The temperature dependence of density is described by a linear function. This approximate description of the medium properties is called the

Boussinesq approximation. Its applicability is limited by rather small temperature differences and linear dimensions of the plate. However, by accounting for the basic part of the density change, the Boussinesq approximation correctly reflects the main specific features of free-convective heat transfer. In this case the work of compression and viscous dissipation of energy are assumed to be negligibly small [30].

The boundary conditions consist of the conditions of fluid clinging to the plate, constant temperature on the plate, symmetry on the symmetry axis, absence of motion and of heating at infinity

$$\frac{\partial\Psi}{\partial y}(x,0) = \frac{\partial\Psi}{\partial x}(x,0) = 0, \quad \Theta(x,0) = 1, \quad 0 < x < 1,$$

$$\frac{\partial^2\Psi}{\partial y^2}(x,0) = \frac{\partial\Psi}{\partial x}(x,0) = \frac{\partial\Theta}{\partial y}(x,0) = 0, \; x < 0, \; x > 1,$$

$$\frac{\partial\Psi}{\partial y} = \frac{\partial\Psi}{\partial x} = \Theta \to 0, \quad r \to \infty, \quad \phi \neq 0. \qquad (2)$$

The basic parameters that characterize the process of free-convective heat transfer are the Grashof and Prandtl numbers. The Prandtl number is determined by the physical parameters of the medium and characterizes the similarity between the vorticity distribution and heat diffusion. Consequently, the most important characteristic of free-convective heat transfer, which is similar to the Reynolds number for the problems of streamlining, is the Grashof number.

To enable a detailed analysis of the leading and trailing edge effects, of the influence of external flow and wake behind a plate on the free-convective heat transfer to be made, it is necessary not only to obtain the solution of the problem, equations (1) and (2), at $Gr \to \infty$, but also to reveal the trends in the solution departure from that for the limited case with the Grashof number deviation from its limiting value. In this case, the parameter perturbations should not be small.

3. BOUNDARY-LAYER INTERACTION WITH THE EXTERNAL FLOW

The boundary-layer interaction with the external flow is usually studied by the method of matched asymptotic expansions [11–13, 31, 32]. When the asymptotic expansions are constructed, the formal limits for the system of equations (1) are investigated, depending on the law of the limit transition as $Gr \to \infty$, and the regions of their applicability are determined [28]. For the temperature and tangential velocity, retaining the order of unity within the whole flow region, there are only two nontrivial limits of the system of equations (1), which determine the trends of motion in the boundary layer and in the external flow. Therefore, an approximate solution of the system of equations (1) is represented in the form of asymptotic expansions which are valid in the boundary layer and in the external flow, respectively:

outer expansion

$$\Psi(x,y;Gr) = \Psi_0(x,y) + Gr^{-1/4}\,\Psi_1(x,y) + ,\ldots,$$
$$\Theta(x,y;Gr) = T_0(x,y) + Gr^{-1/4}\,T_1(x,y) + ,\ldots, \qquad (3)$$

as $Gr \to \infty$, for fixed x, y;

inner expansion

$$\Psi(x,y;Gr) = Gr^{-1/4}\,\Phi_0(x,Y) + Gr^{-1/2}\,\Phi_1(x,Y) + ,\ldots,$$
$$\Theta(x,y;Gr) = \Theta_0(x,Y) + Gr^{-1/4}\,\Theta_1(x,Y) + ,\ldots, \qquad (4)$$

as $Gr \to \infty$, for fixed x; $Y = y\,Gr^{1/4}$.

The boundary-layer free-convective flow induces the external flow, which, in turn, exhibits an inverse effect on the boundary layer, etc. Formally, this is expressed as matching, i.e. the agreement of expansions in the area where the regions of their applicability intersect.

The substitution of expansions (3) and (4) into equations (1) and (2) and a corresponding limit transition determine a number of boundary-value problems for the functions Ψ_i, T_i, Φ_i, Θ_i to be found.

In the zero approximation, the external problem has only a trivial solution ($\Psi_0 \equiv 0$, $T_0 \equiv 0$), while the internal problem is the classical problem of the boundary-layer theory. The similarity transformation

$$\Phi_0(x,Y) = x^{3/4}F_0(\eta),$$
$$\Theta_0(x,Y) = H_0(\eta), \quad \eta = Yx^{-1/4}, \qquad (5)$$

in the case of a semi-infinite plate leads to a boundary-value problem for the system of ordinary differential equations [33]

$$F_0''' + \tfrac{3}{4}F_0''F_0 - \tfrac{1}{2}F_0'F_0' + H_0 = 0,$$
$$H_0'' + \tfrac{3}{4}Pr\,H_0'F_0 = 0,$$
$$F_0(0) = F_0'(0) = F_0'(\infty) = H_0(\infty) = 0, \quad H_0(0) = 1. \qquad (6)$$

The boundary-layer interaction with the external flow is manifested in the boundary conditions for the external problems being formed by the boundary layer, and the boundary conditions on the outer edge of the boundary layer, by the external flow.

In the first approximation, an irrotational external flow is described by the Dirichlet problem for the Laplace equation

$$\nabla^2\Psi_1 = 0,$$
$$\Psi_1(x,0) = x^{3/4}F_0(\infty), \qquad (7)$$
$$x > 0; \quad \Psi_1(x,0) = 0, \quad x < 0.$$

The solution of this problem is given by the Poisson integral for a semi-plane

$$\Psi_1(x,y) = -\sqrt{2}F_0(\infty)r^{3/4}\sin\tfrac{3}{4}(\phi-\pi). \qquad (8)$$

The solution obtained corresponds to an isothermal external flow induced by the displacing effect of the boundary layer. With the aid of the similarity transformation

$$\Phi_1(x,Y) = F_1(\eta), \quad \Theta_1 = x^{-3/4}H_1(\eta), \qquad (9)$$

the system of equations of the first approximation for the boundary layer is reduced to a linear system of differential equations

$$F_1''' + \tfrac{3}{4}F_1''F_0 - \tfrac{1}{4}F_1'F_0' + H_1 = 0,$$
$$H_1'' + \tfrac{3}{4}Pr(F_0H_1)' = 0, \quad (10)$$

subject to the boundary conditions

$$F_1(0) = F_1'(0) = H_1(0) = H_1(\infty) = 0,$$
$$F_1'(\infty) = \tfrac{3}{4}F_0(\infty). \quad (11)$$

Because of the uniqueness of the solution, it follows that

$$H_1(\eta) \equiv 0. \quad (12)$$

The result obtained shows that the first approximation does not influence the heat transfer. This makes it necessary to continue the solution at least up to and including the second approximation [15, 16, 28, 29]. The formulation of the second approximation problems becomes cumbersome, while a small contribution of the second approximation to the solution makes its computation unnecessary. A much greater contribution comes from the eigensolutions satisfying zero boundary conditions. Allowance for the eigensolutions is achieved by the terms of the form

$$c_k\, Gr^{-\lambda_k + 1/4} f_k(\eta) x^{3/4(1-\lambda_k)}, \quad (13)$$

for the stream function and by

$$c_k\, Gr^{-\lambda_k/4}\, g_k(\eta) x^{-3/4\lambda_k}, \quad (14)$$

for the temperature added to the inner expansion. The eigenfunctions f_k and g_k are determined by the boundary-value problems

$$f_k''' + \tfrac{3}{4}f_k''F_0 + (\tfrac{3}{4}\lambda_k - 1)f_k'F_0' + \tfrac{3}{4}(1-\lambda_k)f_kF_0'' + g_k = 0,$$
$$g_k + Pr[\tfrac{3}{4}g_k'F_0 + \tfrac{3}{4}\lambda_k g_k F_0' + \tfrac{3}{4}(1-\lambda_k)f_kH_0'] = 0,$$
$$f_k(0) = f_k'(0) = f_k'(\infty) = g_k(0) = g_k(\infty) = 0. \quad (15)$$

The eigenvalues λ_k are found numerically [15]

$$\lambda_1 = \tfrac{4}{3}, \quad \lambda_2 = 3.189, \quad \lambda_3 = 8.311, \ldots$$

Within the accuracy considered, a substantial role is played only by the first eigensolution which is written out in the explicit form as

$$f_1(\eta) = \tfrac{3}{4}F_0 - \tfrac{1}{4}\eta F_0', \quad g_1(\eta) = -\tfrac{1}{4}\eta H_0'. \quad (16)$$

Physical eigensolutions correspond to the indeterminacy of the position of the longitudinal coordinate origin because of the absence of information on the prehistory of flow in the considered stationary case. In general, the constants c_k cannot be determined within the framework of the method of matched asymptotic expansions.

Summarizing the results obtained we have for the inner expansion

$$\Psi(x, y; Gr) = Gr^{-1/4}x^{3/4}F_0(\eta) + Gr^{-1/2}F_1(\eta)$$
$$+ c_1\, Gr^{-7/12}x^{-1/4}f_1(\eta) + O(Gr^{-3/4}),$$
$$\Theta(x, y; Gr) = H_0(\eta) + Gr^{-1/4}x^{-3/4}H_1(\eta)$$
$$+ c_1\, Gr^{-1/3}x^{-1}g_1(\eta) + O(Gr^{-1/2}). \quad (17)$$

The expressions for the velocity and heat transfer are obtained from the above by differentiation. Equations (17) have non-integrable singularities at $x = 0$, i.e. at the leading edge of the plate. This and the indeterminacy of constants in the eigensolutions make it impossible to regard the solution as fully complete.

4. THE LEADING EDGE EFFECTS

The singularities of the solution constructed are attributed to the discontinuity in the boundary conditions at the leading edge of the plate. The failure to study in detail a flow in the vicinity of the leading edge, within the framework of the asymptotic theory considered, compels one to look for alternative means which would make it possible to account for the effect of the leading edge on free convection at large Grashof numbers. For this purpose, we shall resort to the ideas of the method of deformed coordinates [11–13].

The above description of free convection from a vertical plate was based on the assumption that the boundary layer begins at the leading edge, while the real flow is observed upstream of it. With the aid of a deformed longitudinal coordinate, it is possible to shift the singularities in the internal solution to their true position and to determine more precisely the vertical location of the boundary layer relative to the leading edge. The structure of the boundary-layer interaction with the external flow is preserved in this case. As anticipated, the corrections caused by the flow at the leading edge have been obtained for higher approximations. In conformity with the general ideas of deformation [28, 34, 35], the longitudinal coordinate can be represented in the form of the asymptotic expansion

$$x = X + Gr^{-1/4}f(X, Y) +, \ldots \quad (18)$$

Actually, only the above two terms are used, since the effect of subsequent terms manifests itself in the second and higher order approximations. The deformation is weak in the sense, that the main part of expansion (18) is the very quantity x.

With deformation taken into account, the asymptotic expansions representing the solution take on the form:

outer expansion

$$\Psi(x, y; Gr) = Gr^{-1/4}\Psi_1(X, y) +, \ldots, \Theta(x, y; Gr) \equiv 0, \quad (19)$$

as $Gr \to \infty$, for fixed X, y; $x = X + Gr^{-1/4}f(X, 0)$;

inner expansion

$$\Psi(x,y;Gr) = Gr^{-1/4}\Phi_0(X,Y) + Gr^{-1/2}\Phi_1(X,Y)+\ldots, \quad (20)$$

$$\Theta(x,y;Gr) = \Theta_0(X,Y)+Gr^{-1/4}\Theta_1(X,Y)+\ldots,$$

as $Gr \to \infty$, for fixed X, Y.

The deformation of the longitudinal coordinate does not influence the zero approximation of the boundary layer and the first approximation of the external flow. The deformation makes the boundary conditions to the first approximation of the boundary layer more complex

$$\frac{\partial \Phi_1}{\partial Y}(X,\infty) = \frac{\partial \Psi_1}{\partial y}(X,0)+\frac{\partial f}{\partial Y}\frac{\partial \Phi_0}{\partial X}(X,\infty),$$

just as the whole system of equations in the first approximation becomes more complex. Nevertheless, the similarity solution exists in the form [28, 34, 35]

$$\Phi_1(X,Y) = f\frac{\partial \Phi_0}{\partial X}+F_1(\eta),$$
$$\Theta_1(X,Y) = f\frac{\partial \Theta_0}{\partial X}+X^{-3/4}H_1(\eta), \quad (21)$$

with the quantities F_1 and H_1 being determined by the same boundary-value problem, equations (10) and (11), as before. The condition for the similarity solution conservation allows the specification of the general form of the deformation function

$$f(X,Y) = X^{1/4}\sum_{-\infty}^{\infty} a_n\eta^n. \quad (22)$$

The condition of the deformation function regularity at $x = 0$ and $y = 0$ leads to the simplification

$$f(X,Y) = a_0X^{1/4}+a_1Y. \quad (23)$$

The coefficients a_0 and a_1 are determined by the asymptotic behaviour of the first approximation stream function at the outer edge of the boundary layer [28]

$$a_0 = \frac{4}{3}\frac{F_1(\infty)}{F_0(\infty)}, \quad a_1 = -1. \quad (24)$$

The value of the deformed coordinate at the leading edge of the plate is, accurate to terms of the order of $Gr^{-1/2}$

$$X_0 = (-a_0)^{4/3}Gr^{-1/3}. \quad (25)$$

Since $Gr^{1/3} \sim L$, then $X_0 \sim L^{-1}$. This means that the flow described starts well before the leading edge, and the relative vertical displacement of the boundary layer is the more appreciable, the smaller the length of the plate (Fig. 1). The deformation of the longitudinal coordinate does not exhibit any effect on the first eigensolution and it can be written as

$$f_1(\eta) = c_1X^{1/4}\frac{\partial \Phi_0}{\partial X}, \quad g_1 = c_1X^{1/4}\frac{\partial \Theta}{\partial X}. \quad (26)$$

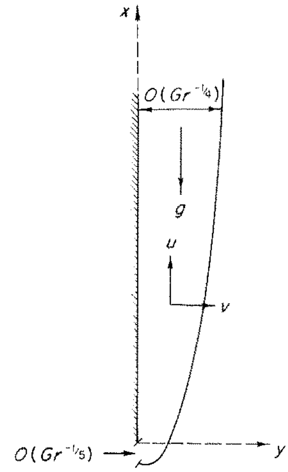

FIG. 1. Schematic of the flow with a deformed longitudinal coordinate.

Comparison between the form of the eigenfunctions, equation (26), and the expression for the first boundary-layer approximation, equation (21), shows that the part of equation (21), which is contributed by deformation, actually contains the first eigensolution. Therefore, the constant in the first eigensolution is determined in terms of the deformation function coefficients [28]

$$c_1 = -2a_0. \quad (27)$$

The coupling between the deformation and eigensolutions seems to be quite natural, since the latter appear because of the absence of detailed information on the flow at the leading edge.

Thus, the deformation of the longitudinal coordinate makes it possible to more accurately describe free convection near a vertical plate with account for the presence of flow induced upstream of its leading edge. The corrections that should be made in the boundary-layer theory appear already in the first approximation. The constant in the eigensolution is determined by the deformation function.

5. FLOW STRUCTURE IN THE VICINITY OF THE TRAILING EDGE

The unique features of flow at the trailing edge of the vertical plate are determined by varying boundary conditions on the line $y = 0$. The no-slip and constant temperature conditions on the plate should pass into the conditions of symmetry on the axial line of the wake. The extension of the trailing edge neighbourhood along x is unknown beforehand and is determined from the analysis of the possible trends of the flow on the basis of the Navier–Stokes and energy equations [28, 36]. This analysis leads to a two-layer structure of boundary-layer flow in the vicinity of the trailing edge (Fig. 2). The outer part of the boundary layer constitutes a flow which is symmetric about the line $X = 1$ and which is

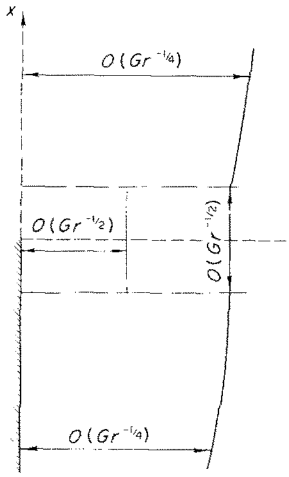

FIG. 2. Flow structure in the vicinity of the trailing edge.

described in the zero approximation by

$$\frac{\partial \Psi_{0L}}{\partial Y_L}\frac{\partial^3 \Psi_{0L}}{\partial X_L^3}-\frac{\partial \Psi_{0L}}{\partial X_L}\frac{\partial^3 \Psi_{0L}}{\partial Y_L \partial X_L^2}=\frac{\partial^4 \Psi_{0L}}{\partial X_L^4},$$

$$\frac{\partial \Psi_{0L}}{\partial Y_L}\frac{\partial \Theta_{0L}}{\partial X_L}-\frac{\partial \Psi_{0L}}{\partial X_L}\frac{\partial \Theta_{0L}}{\partial Y_L}=Pr^{-1}\frac{\partial^2 \Theta_{0L}}{\partial X_L^2}, \tag{28}$$

where

$$T_L = y\,Gr^{1/4}, \quad X_L = (X-1)Gr^{1/2}.$$

In this case the required change in the boundary conditions is not effected, therefore a viscous sublayer is introduced into consideration. The solution continuity dictates the necessity of preserving the higher-order derivatives along both coordinates, as a result of which the system of equations for the zero approximation in a viscous sublayer has the form [28, 36]

$$\nabla^4 \Psi_{0p}+\frac{\partial \Theta_{0p}}{\partial y_p}=0, \quad \nabla^2 \Theta_{0p}=0, \tag{29}$$

where

$$y_p = y\,Gr^{1/2}, \quad x_p = (X-1)Gr^{1/2}.$$

The solution of the second equation of the system (29) is determined directly by virtue of the singularity $\Theta_{0p} \equiv 1$. The flow isothermicity in the viscous sublayer determines the effect of the trailing edge on free-convective heat transfer. Since $\partial \Theta_{0p}/\partial y_p \equiv 0$, then the average heat transfer coefficient decreases by the order $Gr^{-1/4}$, i.e. the trailing edge effect manifests itself no sooner than in the second approximation. The difference in the extents of the effect of the leading and trailing edges on heat transfer is attributed to the trailing edge being covered with a developed boundary layer.

Within the accuracy considered, the study of the higher approximations in the vicinity of the trailing edge seems to be superfluous.

6. WAKE EFFECT BEHIND A PLATE

In the wake region behind a plate, two different flow zones can be distinguished: the near and the far wakes. In the near wake, there occurs the rearrangement of the velocity and temperature from the original profiles, determined by the boundary layer at the plate, to similar profiles. The far wake is characterized by the similarity of the velocity and temperature profiles.

The theoretical [37] and experimental [38] investigations of the near wake in free convection from a vertical plate show that the velocity and temperature profiles in the near wake change rapidly, adjusting to the flow conditions in a fully developed wake. This allows one to ignore the near wake effect on the boundary layer at the plate, since this effect manifests itself not directly but via the external flow.

In the far wake, the inner expansion is presented in the form

$$\Psi(x,y;Gr) = Gr^{-1/4}\Phi_{0c}(x,Y) + Gr^{-1/2}\Phi_{1c}(x,Y)+\ldots, \tag{30}$$

$$\Theta(x,y;Gr) = \Theta_{0c}(x,Y)+Gr^{-1/4}\Theta_{1c}(x,Y)+\ldots.$$

The choice of the similarity variables for the zero approximation in the far wake is dictated by a constant amount of heat in the wake cross-sections

$$\int_0^\infty \frac{\partial \Phi_{0c}}{\partial Y}\Theta_{0c}\,dY = -H_0(0). \tag{31}$$

Whence it follows that

$$\Phi_{0c}(x,Y) = x^{3/5}F_{0c}(\eta),$$

$$\Theta_{0c}(x,Y) = x^{-3/5}H_{0c}(\eta), \quad \eta = Yx^{-2/5}.$$

This same condition provides a nontrivial zero approximation solution of the similarity boundary-value problem in the far wake

$$F'''_{0c}+\tfrac{3}{5}F''_{0c}F_{0c}-\tfrac{2}{5}F'_{0c}F'_{0c}+H_{0c}=0,$$

$$H''_{0c}+\tfrac{3}{5}Pr(H_{0c}F_{0c})'=0, \tag{32}$$

$$F_{0c}(0) = F''_{0c}(0) = H'_{0c}(0) = F'_{0c}(0) = H_{0c}(\infty) = 0.$$

In the presence of the wake, the first approximation for the external flow is described by the Laplace equation

$$\nabla^2 \Psi_1 = 0, \tag{33}$$

with boundary conditions

$$\frac{\partial \Psi_1}{\partial x}(x,0) = 0, \quad x<0,$$

$$\frac{\partial \Psi_1}{\partial x}(x,0) = -\frac{3}{4}x^{-1/4}F_0(\infty), \quad 0<x<1, \tag{34}$$

$$\frac{\partial \Psi_1}{\partial x}(x,0) = -\frac{3}{5}x^{-2/5}F_{0c}(\infty), \quad x>1.$$

The complex potential of the velocity field for the first approximation of the external flow is determined by the Schwarz integral for a semi-plane

$$u_1 - i\, v_1 = -\frac{3}{4} F_0(\infty)\frac{1}{\pi}\int_0^1 \frac{d\lambda}{\lambda^{1/4}(z-\lambda)} - \frac{3}{5} F_{0c}(\infty)\frac{1}{\pi}\int_1^\infty \frac{d\lambda}{\lambda^{2/5}(z-\lambda)}, \quad (35)$$

where $z = x + i\, y$.

As a result, in the first approximation we will have for the boundary layer near a plate [28]

$$\Phi_1(X, Y) = f\frac{\partial \Phi_0}{\partial X} + F_{11}(\eta)\,[\phi_1(x) + \phi_2(x)],$$
$$\Theta_1(X, Y) = f\frac{\partial \Theta_0}{\partial X} + X^{-3/4}H_{11}(\eta)\,[\phi_1(x) + \phi_2(x)]. \quad (36)$$

The form of functions $\phi_1(x)$ and $\phi_2(x)$ are rather cumbersome [28] and therefore are not given here.

Thus, a change in the boundary conditions at the outer edge of the wake, as compared with the conditions at the outer edge of the boundary layer near the plate, leads to a change in the external first approximation problem. In turn, this change manifests itself in the boundary conditions for the internal first approximation problem and leads to the dependence of its solution not only on the similarity variable, but on the distance from the leading edge of the plate.

7. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Free convection is characterized by velocity and temperature distributions in the boundary layer. The resulting expressions for the temperature and longitudinal velocity profiles are

$$\frac{T - T_\infty}{T_w - T_\infty} = \alpha_0 + \alpha_{11}\, Gr_x^{-1/4} + \alpha_{10}\, Gr_x^{-1/4} + \alpha_{4/3}\, Gr_x^{-1/3} + O(Gr_x^{-1/2}),$$
$$\frac{\bar{u}\bar{x}}{\nu\, Gr_x^{1/2}} = \beta_0 + \beta_{11}\, Gr_x^{-1/4} + \beta_{10}\, Gr_x^{-1/4} + \beta_{4/3}\, Gr_x^{-1/3} + O(Gr_x^{-1/2}), \quad (37)$$

where

$$\alpha_0 = H_0(\eta), \quad \alpha_{11} = H_{11}(\eta)\,[\phi_1(x) + \phi_2(x)],$$
$$\alpha_{10} = (a_0 - \eta)\,[-\tfrac{1}{4}\eta H_0'(\eta)], \quad \alpha_{4/3} = \tfrac{1}{2}a_0\eta H_0'(\eta),$$
$$\beta_0 = F_0'(\eta), \quad \beta_{11} = F_{11}'(\eta)\,[\phi_1(x) + \phi_2(x)],$$
$$\beta_{10} = (a_0 - \eta)\,[\tfrac{1}{2}F_0'(\eta) - \tfrac{1}{4}\eta F_0''(\eta)],$$
$$\beta_{4/3} = 2a_0[\tfrac{1}{2}F_0'(\eta) - \tfrac{1}{4}\eta F_0''(\eta)].$$

The above relations have the same structure. In each, the first term corresponds to the boundary-layer theory, the second term is determined by the effect of external flow with allowance for the presence of a wake behind a plate, the third is obtained due to deformation and represents the effects of the leading edge, the fourth corresponds to the first eigensolution. The region of applicability of these equations is determined by the condition $Gr_x > 1$, however, it seems preferable to confine ourselves to a more stringent condition $Gr_x > 10^2$, since the error is estimated only by the order of magnitude.

The comparison of the calculated and experimental [8–10] velocity and temperature profiles is shown in Figs. 3 and 4. For comparative purposes, the curves of the boundary-layer theory are also given. The experimental data supports the theory developed.

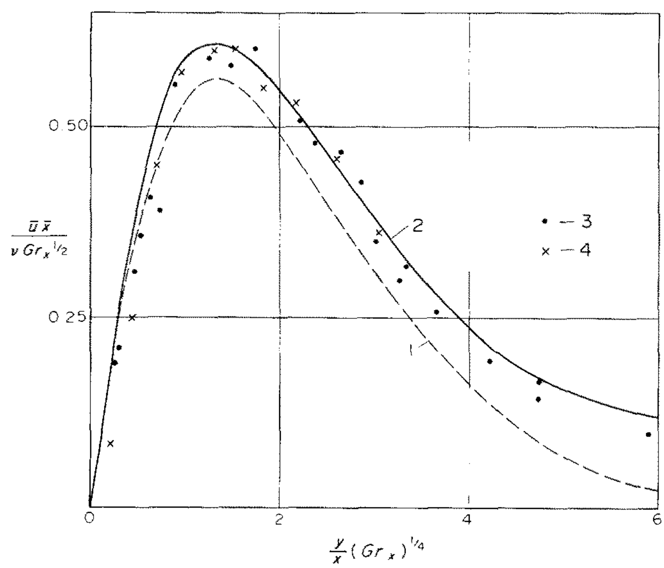

FIG. 3. Velocity distribution in a boundary layer ($Pr = 0.7$): (1) boundary-layer theory; (2) calculation from equation (37), $x = 1/2$, $Gr_x = 10^3$. Experiment: (3) $x = 1/24$, $Gr_x = 3.1 \times 10^3$ [10]; (4) $x = 1/12$, $Gr_x = 5.6 \times 10^3$ [8].

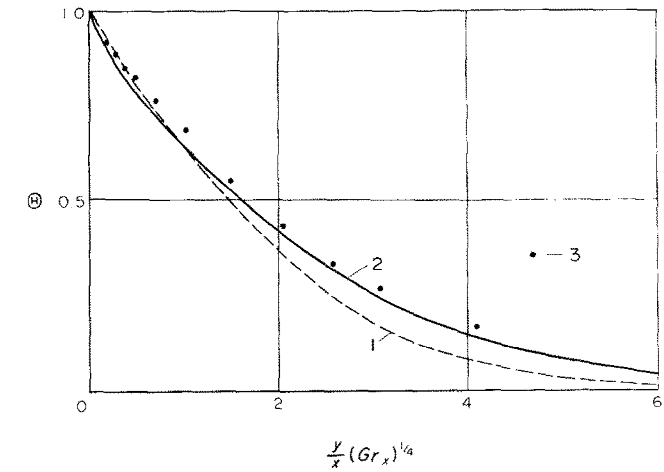

FIG. 4. Temperature distribution in a boundary layer ($Pr = 0.7$): (1) boundary-layer theory; (2) calculation from equation (37), $x = 1/12$, $Gr_x = 10^2$; (3) experiment, $x = 1/24$, $Gr_x = 6.8 \times 10^2$ [8].

The theoretical velocity and temperature profiles reflect the overall effect of different factors. The leading edge effect leads to smaller velocity and temperature values than those from boundary-layer theory. The eigensolutions furnish a positive correction for both the velocity and temperature, being in the latter case an absolute value larger, than in the former. The effect of external flow leads to an increase in the velocity, including the case when the wake behind a plate is taken into account. The temperature in this case somewhat decreases near the plate surface and increases with distance from it.

With an increasing Grashof number the theoretical profiles come close to the boundary-layer theory curves.

An important feature for practical application is the determination of the heat transfer characteristics. The structure of the expression for the local heat transfer coefficient is similar to equation (37), namely

$$Nu_x = \gamma_0 Gr_x^{1/4} + \gamma_{11} + \gamma_{10} + \gamma_{4/3} Gr_x^{-1/12} + O(Gr_x^{-1/4}), \quad (38)$$

where

$$\gamma_0 = -H_0'(0), \quad \gamma_{11} = -H_{11}'(0)[\phi_1(x) + \phi_2(x)],$$

$$\gamma_{10} = \tfrac{1}{4}a_0 H_0'(0), \quad \gamma_{4/3} = -\tfrac{1}{2}a_0 H_0'(0).$$

The length-average heat transfer coefficient is determined by integration

$$Nu_L = \int_{X_0}^{x_1} \frac{Nu_x}{X}\, dX, \quad (39)$$

at

$$X_0 = (-a_0)^{4/3} Gr^{-1/3}, \quad x_1 = 1 + a_0 Gr^{-1/4} - Gr^{-1/2}.$$

A comparison between the theoretical and experi-

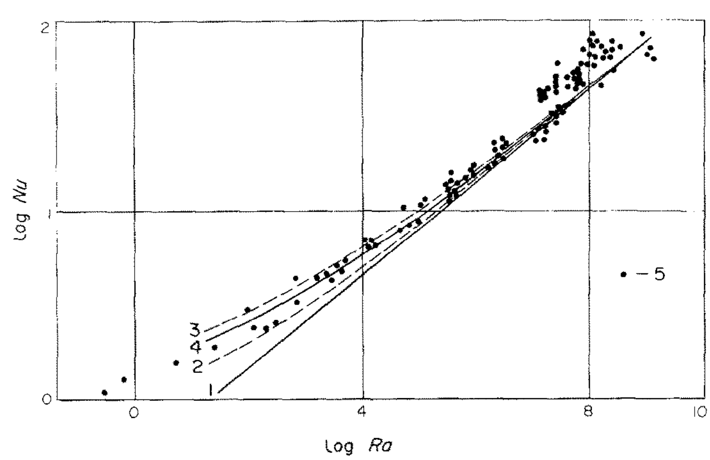

FIG. 5. Comparison between calculation and experiments on the basis of the mean heat transfer coefficient: (1) boundary-layer theory; (2) with external flow taken into account; (3) with the leading edge effect taken into account; (4) with the trailing edge and wake effects taken into account; (5) experiment [1].

mental data [1] on heat transfer is presented in Fig. 5. Curve 1 corresponds to the classical boundary-layer theory; curve 2, to its modernization when the boundary-layer interaction with the external flow is taken into account; curve 3 allows for the leading edge effect; and curve 4 allows for the trailing edge and wake effects. Thus, the leading edge effect and the interaction with the external flow lead to an increase in heat transfer, while the effect of the trailing edge and of the wake lead to a decrease, which does not balance out the above increase. The resulting curve gives an adequate description of the experimental data with decreasing Grashof numbers.

The largest departure from the classical boundary-layer theory is observed at the smallest Grashof numbers. Quantitatively, it amounts to 45% ($Gr_L = 10^2$).

The comparison between the theoretical and experimental results shows that the asymptotic theory developed provides a correct qualitative picture of free-convective heat transfer near a vertical plate with account for the specific features of the phenomenon and gives a good qualitative approximation to an accurate solution.

8. FREE CONVECTION AT SMALL PRANDTL NUMBERS

Uniform heating of a heat transfer surface serves as a convenient model for the formulation of free convection problems, which however, can hardly be realized in practice. A typical and practically most often realizable case is that when a constant heat flux is prescribed on the wall. In the majority of cases the investigations of free-convective heat transfer with a constant heat flux on the wall have been carried out for small Prandtl numbers [39–43].

The description of free convection at small Prandtl numbers presupposes the analysis of interaction between the thermal and velocity layers.

The basic system of equations for free convection at small Prandtl number is presented in the form [28]

$$\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\Psi)-\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\Psi) = Gr^{*-2/5}\,Pr^{1/5}\,\nabla^4\Psi+Gr^{*1/5}\,Pr^{2/5}\frac{\partial\Theta}{\partial y}, \quad (40)$$

$$\frac{\partial\Psi}{\partial y}\frac{\partial\Theta}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial\Theta}{\partial y} = Gr^{*-2/5}\,Pr^{-4/5}\,\nabla^2\Theta.$$

The boundary conditions remain the same as in the isothermal case except for the thermal condition on the wall

$$\frac{\partial\Theta}{\partial y}(x,0) = -1, \quad 0<x<1. \quad (41)$$

The analysis of the possible trends in the flow at $Gr^*\,Pr^2 \to \infty$ shows [28], that two layers—thermal and velocity ones—can be distinguished in the boundary layer (Fig. 6). However, the presence of the velocity layer does not influence the heat transfer, since the heat flux magnitude does not change on passing through this layer. The effect of the dynamic sublayer on the velocity profiles can be taken into account by preserving the higher derivative in the equations for the thermal layer [28]. As a result, it is sufficient to limit ourselves to the study of motion in the thermal layer and in the external flow.

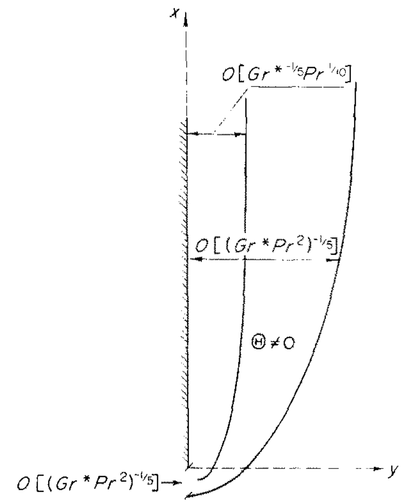

FIG. 6. Scheme of the flow at small Prandtl numbers.

The solution is presented in the form of asymptotic expansions:

outer

$$\Psi(x,y;Gr^*,Pr) = (Gr^*\,Pr^2)^{-1/5}\Psi_{11}(x,y)+,\ldots,$$
$$\Theta(x,y;Gr^*,Pr) = 0, \quad (42)$$

as $Gr^*\,Pr^2 \to \infty$, for fixed x, y;

inner

$$\Psi(x,y;Gr^*,Pr) = (Gr^*\,Pr^2)^{-1/5}\Phi_0(X,Y) +(Gr^*\,Pr^2)^{-2/5}\Phi_1(X,Y)+,\ldots, \quad (43)$$
$$\Theta(x,y;Gr^*,Pr) = (Gr^*\,Pr^2)^{-1/5}\Theta_0(X,Y)+,\ldots,$$

as $Gr^*\,Pr^2 \to \infty$, for fixed X; $Y = y\,Gr^{*1/5}\,Pr^{2/5}$.

The process of solution construction is quite similar to that described above for the solution of the problem of free convection at the isothermal surface, including the deformation of the longitudinal coordinate, the study in the vicinity of the trailing edge and of the wake behind the plate [28, 44].

The expressions for the velocities and temperatures in the boundary layer for the heat transfer coefficient have the same structure as before

$$\frac{\lambda(T-T_\infty)}{q_w\bar{x}} = \alpha_0(Gr_x^*\,Pr^2)^{-1/5}+\alpha_{11}(Gr_x^*\,Pr^2)^{-2/5} +\alpha_{10}(Gr_x^*\,Pr^2)^{-2/5}+\alpha_{5/4}(Gr_x^*\,Pr^2)^{-9/20} +O[(Gr_x^*\,Pr^2)^{-3/5}], \quad (44)$$

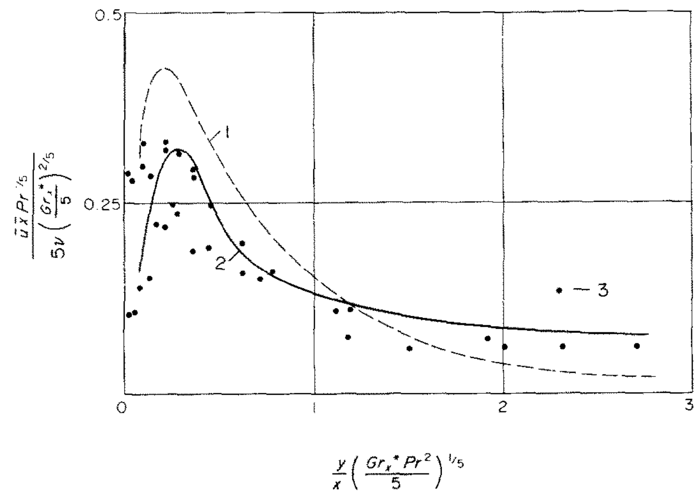

FIG. 7. Velocity distribution at small Prandtl numbers: (1) boundary-layer theory; (2) calculation from equation (45), $x = 7/12$, $Gr_x^* = 10^6$; (3) experiment, $x = 0.6$, $Gr_x^* = 4\times10^7$ [41].

$$\frac{\bar{u}x\,Pr^{1/5}}{5\nu(Gr_x^*/5)^{1/5}} = \beta_0+\beta_{11}(Gr_x^*\,Pr^2)^{-1/5} +\beta_{10}(Gr_x^*\,Pr^2)^{-1/5}+\beta_{5/4} \times(Gr_x^*\,Pr^2)^{-1/4}+O[(Gr_x^*\,Pr^2)^{-2/5}], \tag{45}$$

$$\frac{Nu_x}{(Gr_x^*\,Pr^2)^{1/5}} = \gamma_0+\gamma_{11}(Gr_x^*\,Pr^2)^{-1/5} +\gamma_{10}(Gr_x^*\,Pr^2)^{-1/5}+\gamma_{5/4} \times(Gr_x^*\,Pr^2)^{-1/4}+O[(Gr_x^*\,Pr^2)^{-2/5}], \tag{46}$$

with a respective change in the coefficients $\alpha_i, \beta_i, \gamma_i$ [28]. The comparison between the theoretical and experimental results [39–41] is given in Figs. 7–9. In the case considered, the effect of the leading edge leads to negative corrections to be used in the classical boundary-layer theory for both the velocity and temperature. The interaction of the boundary layer with the external flow somewhat weakens this effect for the velocity and increases the effect for the temperature. The eigensolutions in both cases give positive corrections which do not compensate for the leading edge effect. The resultant correction turns out to be negative for both the temperature and velocity inside the boundary layer. At the outer edge of the boundary layer the interaction with the external flow increases the longitudinal velocity.

Thus, a change in the boundary condition for the temperature on the plate leads to substantially different trends in free-convective heat transfer which the boundary-layer theory fails to account for.

The comparison of theoretical and experimental data on heat transfer indicates a substantial (up to 50% at $Gr_x^*\,Pr^2 = 10^{-2}$) deviation from the results of the boundary-layer theory which is proved experimentally.

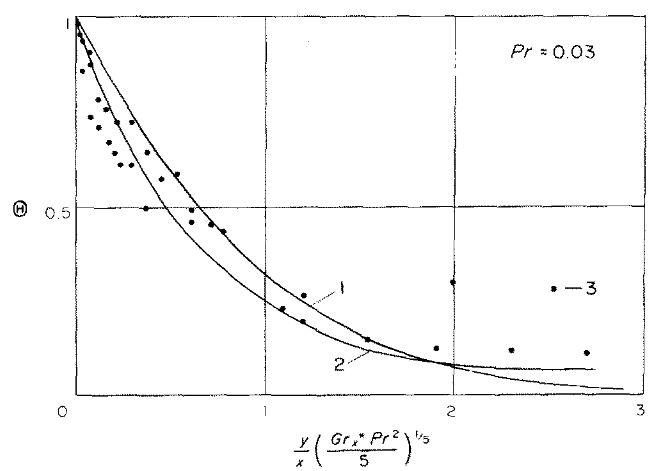

FIG. 8. Temperature distribution at small Prandtl numbers: (1) boundary-layer theory; (2) calculation from equation (44), $x = 7/12$, $Gr_x^* = 10^6$; (3) experiment, $x = 0.6$, $Gr_x^* = 4\times10^7$ [41].

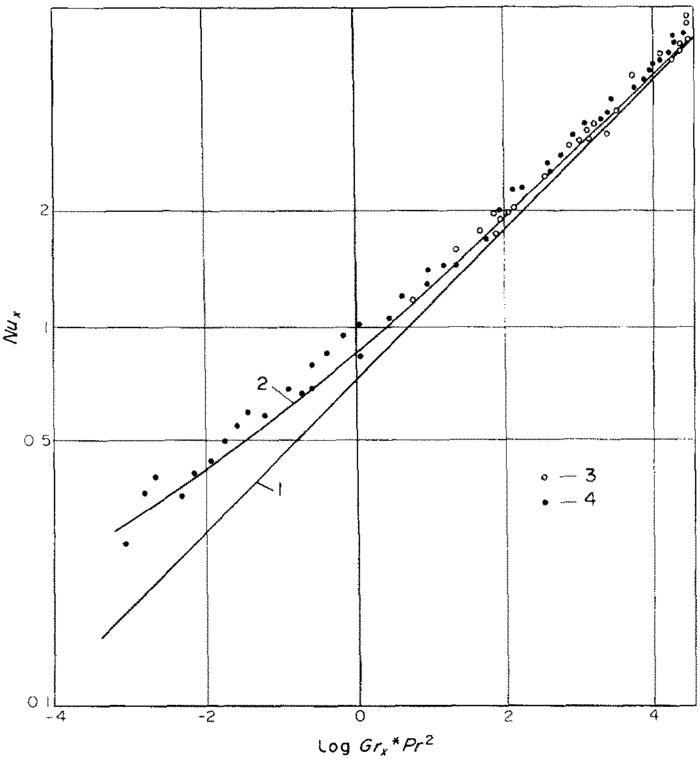

FIG. 9. Comparison between calculations and experiments based on the local value of heat transfer: (1) boundary-layer theory; (2) equation (46); (3) ref. [39]; (4) ref. [40].

9. CONCLUSION

The asymptotic theory based on the methods of singular perturbations and developed for the solution of the Navier–Stokes and energy equations has made it possible to obtain a general description of free convection near a vertical finite plate.

The theoretical description obtained, which takes into account the effects of the leading and trailing edges, interaction with the external flow and wake behind a plate, has revealed the dependence of the trends of motion on the boundary conditions for the temperature on the plate and made it possible to calculate the quantitative characteristics of the phenomenon. Comparison with the experiments has proved the validity of the theory developed.

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CONVECTION NATURELLE LAMINAIRE SUR UNE PLAQUE VERTICALE

Résumé—On considère une solution asymptotique des équations de Navier–Stokes et de l'énergie pour le problème de la convection naturelle sur une plaque finie verticale. L'interaction de la couche limite avec l'écoulement externe est étudiée par la méthode des développements asymptotiques. L'effet du bord d'attaque est analysé à l'aide d'une coordonnée longitudinale déformée. La structure de l'écoulement et le transfert de chaleur au voisinage du bord de fuite sont étudiés par un modèle à deux couches limites. On trouve que l'effet de sillage sur la couche limite se manifeste à travers l'écoulement externe. Les résultats de la résolution sont vérifiés par des expériences sur les profils de vitesse et de température et sur les coefficients de transfert thermique, à la fois dans le cas des conditions aux limites de première espèce et dans le cas d'un flux constant.

LAMINARE FREIE KONVEKTION AN EINER SENKRECHTEN PLATTE

Zusammenfassung—Es wird eine asymptotische Lösung der Navier–Stokes- und der Energie-Gleichungen für das Problem der freien Konvektion an einer senkrechten endlichen Platte behandelt. Die Wechselwirkung der Grenzschicht und der äußeren Strömung wird mit der Methode der angepaßten asymptotischen Entwicklungen untersucht. Die Strömungsstruktur und der Wärmeübergang in der Umgebung der Plattenhinterkante werden im Gültigkeitsbereich des Zwei-Schichten-Grenzschicht-Modells untersucht. Es stellt sich heraus, daß der Nachlaufeffekt auf die Grenzschicht der Platte durch die äußere Strömung deutlich wird. Die theoretischen Ergebnisse werden durch experimentell ermittelte Geschwindigkeits- und Temperaturprofile und Wärmeübergangskoeffizienten bekräftigt, sowohl für die Randbedingungen 1. Art an der Platte wie auch für den Fall konstanter Wärmestromdichte.

ЛАМИНАРНАЯ СВОБОДНАЯ КОНВЕКЦИЯ НА ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ

Аннотация—Рассматривается асимптотическое решение уравнений Навье–Стокса и энергии в задаче свободной конвекции около вертикальной пластины конечной длины. Взаимодействие пограничного слоя с внешним течением изучено методом сращиваемых асимптотических разложений. Влияние передней кромки проанализировано при помощи деформирования продольной координаты. Структура течения и теплообмена в окрестности задней кромки пластины исследована в рамках двухслойной модели пограничного слоя. Установлено, что влияние следа на пограничный слой около пластины проявляется через внешнее течение. Результаты решения подтверждены сравнением с экспериментальными данными для профилей скорости и температуры и для коэффициентов теплоотдачи как в случае граничных условий первого рода для пластины, так и в случае постоянства теплового потока.